

## ON ESTIMATION OF DISTRIBUTION OF TEMPERATURE IN VENTILATED FACADES. AN ANALYTICAL APPROACH FOR PROGNOSIS

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**Abstract.** In this paper distribution of temperature between the wall of a building and the facing of the facade with account possible native convection air movement has been analyzed. An analytical approach for analysis of the distribution to increase of predictability of the heat transport has been introduced.

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**Keywords:** ventilated facade, distribution of temperature, native convective of air, analytical approach for modeling.

**AMS Subject Classification:** 74B05, 39A21.

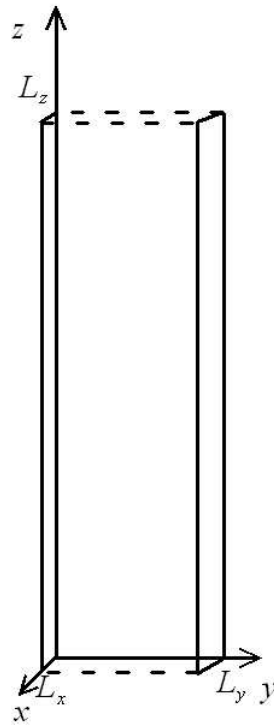
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## 1 Introduction

In modern building construction increasing of interest in the manufacturing of ventilated facades, which are used as decorative construction and as additional thermal and wind defending, could be found. Insulation, which adjoin to the wall of the building from the effects of the environment, could be defended by ventilated facades. In the considered case moisture into the atmosphere will be removed by presence of convective movements of air. After that one can find maintenance of insulation in a state of low humidity. It should be noted, that large convective currents reduce the heat insulating of coating. In this situation development of models for describing the processes occurring during facade ventilation (convection, heat transfer, ...) attracted an interest. Now a series of computational works (Broujerdian & Kazemi, 2016; Alipour, 2016; Polus & Szumigala, 2016; Long et al., 2017; Broujerdian et al., 2018; Argilaga et al., 2019; Zhao et al., 2019; Li et al., 2019) have already been published. However development of existing models and methods for their analysis still attracted an interest now. Structure of the considered ventilated facade shown in Fig. 1. In this figure the  $Oxz$  plane coincides with a building wall, which has a temperature  $T_{in}$ . This temperature could be changed with time (changing of temperature in the room). Outer wall of the facade (at the distance  $L_y$  from the internal wall with room) with the ambient temperature  $T_{ext}$  could be also changed with time with changing of environment. Main aim of this paper is estimation of distribution of temperature inside of the ventilated facade with account of the natural convection, which generated due to inhomogeneity of the distribution of temperature. An accompanying aim of this paper is introduction of an analytical approach for analysis of the distribution to increase of predictability of the heat transport.



**Figure 1:** Structure of considered ventilated facade

## 2 Method of solution

To solve the considered aims spatial-temporal distribution of temperature in the considered ventilated facade has been calculated. The required distribution of temperature has been calculated by solving the following boundary value problem

$$c \frac{\partial T(x, y, z, t)}{\partial t} = \text{div} \{ \lambda \cdot \text{grad} [T(x, y, z, t)] - \vec{v}(x, y, z, t) \cdot c \cdot \rho \cdot T(x, y, z, t) \}. \quad (1)$$

Here  $c$  is the heat capacity of the system;  $T(x, y, z, t)$  is the spatio-temporal distribution of temperature;  $x, y, z$  are the spatial coordinates;  $t$  is the time;  $\rho$  is the spatio-temporal distribution of the concentration of particles of air;  $\lambda$  is the coefficient of thermal conductivity, the value of which is determined by the ratio:  $\lambda = \bar{v} \bar{l} c_v \rho / 3$ , where  $\bar{l}$  is the mean free path of gas molecules between collisions,  $\bar{v}$  is the modulus of the root-mean-square velocity of the gas molecules, which equal to  $\bar{v} = \sqrt{2kT/m}$ ,  $k$  is the Boltzmann constant,  $m$  is the mass of the molecule. It is known that the overwhelming majority in the composition of air of atmosphere is nitrogen. In this situation we neglect by other components of air of atmosphere in comparison with nitrogen. Boundary and initial conditions for Eq. (1) could be written as

$$\begin{aligned} T(0, y, z, t) = T(L_x, y, z, t) = T(x, y, 0, t) = T(x, y, L_z, t) = T(x, L_y, z, t) = T_{ext}(t), \\ T(x, 0, z, t) = T_{in}(t), T(x, y, z, t) = T_{ext}(0) = T_0. \end{aligned} \quad (2)$$

Here  $T_{ext}(t)$  is the ambient temperature,  $T_{in}(t)$  temperature of building wall. Taking into account dependence of the coefficient of thermal conductivity and the root-mean-square velocity of movement of gas molecules on temperature leads to Eq. (1) to the following form

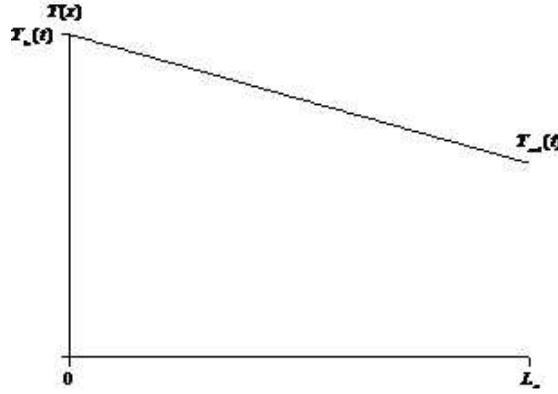
$$c \frac{\partial T(x, y, z, t)}{\partial t} = \frac{\bar{l} c_v \rho}{9} \sqrt{\frac{8k}{m}} \left[ \frac{\partial^2 T^{3/2}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 T^{3/2}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 T^{3/2}(x, y, z, t)}{\partial z^2} \right] -$$

$$\begin{aligned}
 & -c\sqrt{\frac{8k}{9m}} \frac{\partial [T^{3/2}(x, y, z, t) \cdot \rho]}{\partial x} - c\sqrt{\frac{8k}{9m}} \frac{\partial [T^{3/2}(x, y, z, t) \cdot \rho]}{\partial y} - \\
 & -c\sqrt{\frac{8k}{9m}} \frac{\partial [T^{3/2}(x, y, z, t) \cdot \rho]}{\partial z}, \tag{1a}
 \end{aligned}$$

Now this equation has been solved by method of averaging functional corrections (Sokolov, 1955) with a decreased quantity of iteration steps (Pankratov, 2007). Framework this method initial approximation of temperature has been choose as solution of this boundary-value problem without accounting of transient processes:

$$T_0(y, t) = [T_{int} - T_{ext}(t)](1 - y/L_y) + T_{ext}(t). \tag{3}$$

This ratio has been shown in Fig. 2. This relation does not depend on the  $x$  and  $z$  coordinates due to the symmetry of the system.



**Figure 2:** Structure of initial distribution of temperature

Relation (3) has been substituted into the right-hand side of equation (1a). The Substitution leads to obtaining of the first-order approximation of the considered temperature in the following form

$$\begin{aligned}
 \frac{\partial T_1(x, y, z, t)}{\partial t} &= \frac{T_{in}(t) - T_{ext}(t)}{L_y} \sqrt{\frac{2k}{m}} \sqrt{[T_{in}(t) - T_{ext}(t)] \left(1 - \frac{y}{L_y}\right) + T_{ext}(t)} - \\
 & - \frac{\bar{l}c_v\rho}{3cL_y} \sqrt{\frac{k}{2mL_y}} [T_{ext}(t) - T_{in}(t)]^{3/2} \left/ \sqrt{[T_{in}(t) - T_{ext}(t)] \left(1 - \frac{y}{L_y}\right) + T_{ext}(t)} \right. \tag{4}
 \end{aligned}$$

Integration of this equation on time leads to the following result

$$\begin{aligned}
 T_1(y, t) &= \frac{1}{L_y} \sqrt{\frac{k}{m}} \int_0^t [T_{in}(\tau) - T_{ext}(\tau)] \sqrt{[T_{in}(\tau) - T_{ext}(\tau)] \left(1 - \frac{y}{L_y}\right) + T_{ext}(\tau)} d\tau - \\
 & - \frac{\bar{l}c_v\rho}{3cL_y} \sqrt{\frac{k}{2mL_y}} \int_0^t \frac{[T_{ext}(\tau) - T_{in}(\tau)]^{3/2} d\tau}{\sqrt{[T_{in}(\tau) - T_{ext}(\tau)] \left(1 - \frac{y}{L_y}\right) + T_{ext}(\tau)}}. \tag{4a}
 \end{aligned}$$

The second-order approximation of temperature could be obtained by using standard iterative procedure, i.e. by replacement of the considered temperature on the sum of not yet known average value of required approximation ( $\alpha_2$  in our case) and approximation with previous order

and the average value  $\alpha_2$ , i.e.  $T(y, t) \rightarrow \alpha_2 + T_1(y, t)$ . Substitution of this sum into the right side of Eq. (4a) leads to the following result

$$c \frac{\partial T_2(y, t)}{\partial t} = \frac{\bar{l}c_v\rho}{3} \sqrt{\frac{2k}{m}} \frac{\partial}{\partial y} \left[ \sqrt{\alpha_2 + T_1(y, t)} \frac{\partial T_1(y, t)}{\partial y} \right] - c \sqrt{\frac{2k}{m}} \sqrt{\alpha_2 + T_1(y, t)} \frac{\partial T_1(y, t)}{\partial y}. \quad (5)$$

Integration of this equation on time leads to the following result

$$T_2(y, t) = \frac{\bar{l}c_v\rho}{3A} \sqrt{\frac{2k}{m}} \frac{\partial}{\partial y} \left[ \int_0^t \sqrt{\alpha_2 + T_1(y, \tau)} \frac{\partial T_1(y, \tau)}{\partial y} d\tau \right] - \sqrt{\frac{2k}{m}} \int_0^t \sqrt{\alpha_2 + T_1(y, \tau)} \frac{\partial T_1(y, \tau)}{\partial y} d\tau. \quad (5a)$$

Approximations of temperature with higher orders could be obtained by using analogous procedure, i.e. by replacement  $T(y, t)$  on  $\alpha_n + T_n - 1(y, t)$  ( $n$  is the order of required approximation) in the right side of Eq. (1a).

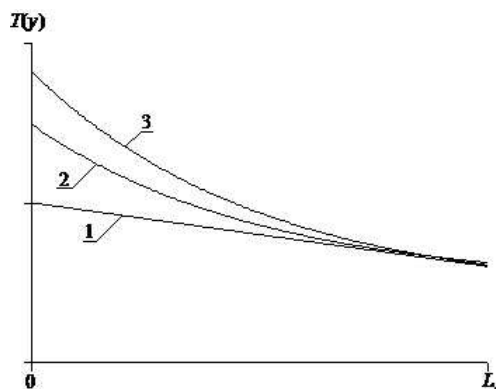
In this paper the second-order approximation of distribution of temperature framework method of averaging functional corrections has been calculated. The approximation is usually enough good approximation for obtaining of qualitative results and obtaining some quantitative results. The results of analytical calculations were verified by comparing them with the results of numerical simulation.

### 3 Discussion

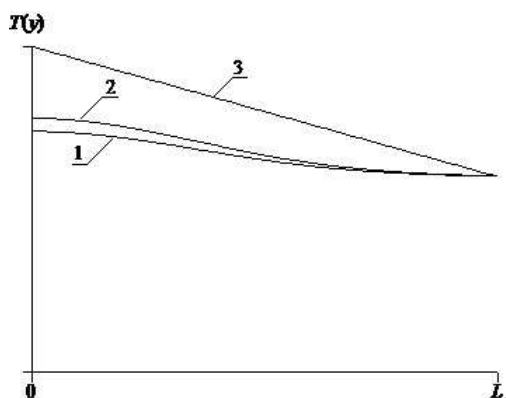
In this section heat transfer in a ventilated facade by calculated in previous section relations has been analyzed. Figs. 3 and 4 show distributions of temperature with variation in time of temperature of wall of building. Dependences of distribution of temperature on time variation of the temperature of the building wall and external boundary of facade were presented on Figs. 3 and 4. In this situation fixed temperatures of external boundary of facade were considered. Fig. 3 show distribution of temperature in facade for increased temperature of the building wall. Increasing of number of curves corresponds to increasing of the temperature. Fig. 4 show distribution of temperature in facade for decreased temperature of the building wall. Increasing of number of curves corresponds to decreasing of the temperature. The obtained distributions of temperature qualitatively coincides with analogous experimental results (Bodrov & Smykov, 2007). The obtained distributions of temperature more precisely in comparison with analogous distributions of temperature without nonlinearity. In this situation we obtain more adequate model of heat transport with possibility to calculated results with higher speed in comparison with recently introduced approaches (Sokolov, 1955).

### 4 Conclusion

An analytical approach for analyzing the assessment of the temperature field distribution between wall of building and external boundary of facade with account possible convective air movement, possible temperature changes on the walls of the ventilation facade and nonlinearity of heat transfer was introduced. Based on this approach distribution of temperature in a ventilation facade has been analyzed. Based on this analysis we obtain, that the approach give a possibility to analyze more adequate model of heat transport with possibility to calculated results with higher speed in comparison with recently introduced approaches.



**Figure 3:** Increase of curve numbers corresponds to increasing temperature of the building wall



**Figure 4:** Increase of curve numbers corresponds to decreasing temperature of the building wall

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